

heat transfer rate time histories provide information on the time of arrival of the acceleration gas-test gas interface, whereas shock standoff distance time histories do not. This implies the shock standoff distance is not as sensitive to change in flow conditions as pressure and heat transfer rate for these expansion tube tests. Heat transfer results revealed flow establishment for helium test gas is more rapid than for air test gas. The present measurements demonstrate that quasi-steady flow exists about relatively large, blunt models during two-thirds of the approximate 250 μ s expansion tube test period.

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Technical Comments

Comment on "A New Method of Solution of the Eigenvalue Problem for Gyroscopic Systems"

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MEIROVITCH¹ considered the following eigenvalue problem governing the oscillation of gyroscopic systems

$$NX + GX = 0 \quad (1)$$

where λ is the eigenvalue, X the corresponding eigenvector, I and G are even-order real nonsingular matrices, the former symmetric and the latter skew-symmetric. By means of the nonsingular transformation $X = ABY$, with A being the orthogonal matrix which diagonalizes I , and B , the diagonal matrix which normalizes the diagonalized I , Eq. (1) is transformed to

$$\lambda B^T A^T I A B Y + B^T A^T G A B Y = \lambda Y - P Y = 0 \quad (2)$$

where $P = -B^T A^T G A B$ is a skew-symmetric matrix. Thus it is seen that the eigenvalue problem considered is really equivalent to a standard eigenvalue problem for a skew-symmetric matrix P . Traditionally skew-symmetric eigenvalue problems have not been of much interest, although results

such as the eigenvalues are conjugate imaginary pairs, the eigenvectors are orthogonal, etc., are reasonable well-known.^{2,4} Since efficient algorithms for computing the eigenvalues and eigenvectors of real symmetric matrices are available,^{2,5} the skew-symmetric eigenvalue problem may be solved by noting if λ and X are eigenvalue and eigenvector of P ; i.e., $PX = \lambda X$, then

$$P^2 X = P(PX) = P(\lambda X) = \lambda PX = \lambda^2 X \quad (3)$$

That is, λ^2 and X are eigenvalue and eigenvector of P^2 , which is a symmetric matrix. A slight complication exists because P^2 has a two-fold multiplicity of all its eigenvalues, and for each eigenvalue there exists a two-dimensional subspace in which every vector is an eigenvector. This ambiguity may be resolved by noting if X_1 and X_2 are a pair of orthonormal eigenvectors of P^2 , associated with the eigenvalue λ^2 , then the corresponding eigenvectors of P are $X = \alpha X_1 \pm i\alpha X_2$. Many of the results and derivations in Ref. 1 follow immediately from the previous result. It may also be pointed out that the transformation from Eqs. (14) and (17) to Eqs. (20) and (22) in Ref. 1 has been programed as Subroutine NROOT in the IBM Scientific Subroutine Package.

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